

$$\int (3-4t)\sqrt{2t-5} dt$$

$$\textcircled{4} \quad u = 2t-5$$

$$du = 2 dt$$

$$\frac{1}{2} du = dt$$

$$t = \frac{u+5}{2}$$

$$3-4t = 3-4\left(\frac{u+5}{2}\right)$$

$$= -2u-7$$

$$\textcircled{2} \quad \frac{1}{2} \int (-2u-7)\sqrt{u} du \textcircled{5}$$

$$= \int \left(-u^{\frac{3}{2}} - \frac{7}{2}u^{\frac{1}{2}}\right) du$$

$$= -\frac{2}{5}u^{\frac{5}{2}} - \frac{7}{2}\left(\frac{2}{3}\right)u^{\frac{3}{2}} + C \textcircled{4}$$

$$= -\frac{2}{5}(2t-5)^{\frac{5}{2}} - \frac{7}{3}(2t-5)^{\frac{3}{2}} + C$$

$$\textcircled{3}$$
$$\textcircled{2}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \theta^2 \csc \theta \, d\theta$$

$\theta^2 \csc \theta$ IS DISCONTINUOUS

① $\theta = 0$

SO FTC DOESN'T APPLY

②

③

$$\int \frac{14 \sin 8y - 21e^{2y}}{\sqrt[3]{6e^{2y} + \cos 8y}} dy$$

$$\textcircled{4} \quad u = 6e^{2y} + \cos 8y$$

$$du = (12e^{2y} - 8 \sin 8y) dy$$

$$= 4(3e^{2y} - 2 \sin 8y) dy$$

$$-\frac{7}{4} du = -7(3e^{2y} - 2 \sin 8y) dy$$

$$= (14 \sin 8y - 21e^{2y}) dy$$

$$= \underbrace{-\frac{7}{4}}_{\textcircled{4}} \int \underbrace{\frac{1}{\sqrt[3]{u}}}_{\textcircled{3}} du$$

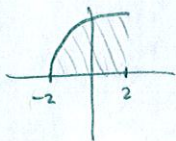
$$= -\frac{7}{4} \left(\frac{3}{2} \right) \underbrace{u^{\frac{2}{3}}}_{\textcircled{4}} + C$$

$$= -\frac{21}{8} \underbrace{(6e^{2y} + \cos 8y)^{\frac{2}{3}}}_{\textcircled{3}} + \underbrace{C}_{\textcircled{2}}$$

$$\int_{-2}^2 (3\sqrt{16-(x-2)^2} - 2x^2 \sin x) dx$$

$$= \underbrace{3 \int_{-2}^2 \sqrt{16-(x-2)^2} dx - \int_{-2}^2 2x^2 \sin x dx}_{(2)}$$

$y = \sqrt{16-(x-2)^2}$
 $(x-2)^2 + y^2 = 16$
CIRCLE RADIUS 4
CENTER (2,0)



$$\underbrace{2(-x)^2 \sin(-x)}_{(2)} = -2x^2 \sin x$$

(2) ODD, CONTINUOUS (2)
INTEGRAL = 0 (2)

$$= \underbrace{3 \cdot \frac{1}{4} \pi (4)^2}_{(3)}$$

$$= \underbrace{12\pi}_{(2)}$$

If $g(x) = \int_{x^2}^{2x} \ln(2+t^3) dt$, find $g''(1)$.

SCORE: ____ / 15 PTS

$$g(x) = \int_{x^2}^{2x} \ln(2+t^3) dt = \int_0^{2x} \ln(2+t^3) dt - \int_0^{x^2} \ln(2+t^3) dt$$

$$g'(x) = \ln(2+(2x)^3) \cdot 2 - \ln(2+(x^2)^3) \cdot 2x = \underbrace{2 \ln(2+8x^3)} - \underbrace{2x \ln(2+x^6)}$$

$$g''(x) = \frac{2 \cdot 24x^2}{2+8x^3} - \underbrace{2 \ln(2+x^6)}_{\textcircled{1}} - \frac{2x \cdot 6x^5}{2+x^6}$$

$$g''(1) = \frac{48}{10} - 2 \ln 3 - \frac{12}{3}$$

$$= \underbrace{\frac{4}{5} - 2 \ln 3}$$

② EACH EXCEPT AS INDICATED

The table gives the acceleration of a car (in meters/minute²) at various times (in minutes).

SCORE: ____ / 15 PTS

At time $t = 2$, the velocity of the car was 7 meters/minute.

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13
$a(t)$	3	8	6	5	2	4	7	9	10	13	11	12	15	14

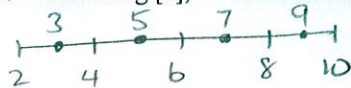
- [a] Write an expression involving an integral for the velocity of the car at $t = 10$.

$$v(10) - v(2) = \int_2^{10} a(t) dt$$

$$v(10) = \underbrace{7}_{(2)} + \underbrace{\int_2^{10} a(t) dt}_{(5)}$$

- [b] Estimate the velocity of the car at $t = 10$ using [a], 4 subintervals and midpoints. Specify the units of your answer.

$$\Delta t = \frac{10-2}{4} = 2$$



$$7 + (a(3) + a(5) + a(7) + a(9))\Delta t = 7 + \underbrace{(5 + 4 + 9 + 13)}_{(4)} \underbrace{(2)}_{(2)}$$

$$= \underbrace{69}_{(1)} \text{ m/min } (1)$$

Find $f(t)$ and a such that $1 + \int_a^x \ln(f(t)) dt = \frac{2}{x^3}$.

SCORE: ____ / 15 PTS

$$\frac{d}{dx} \left(1 + \int_a^x \ln f(t) dt \right) = \frac{d}{dx} \left(\frac{2}{x^3} \right)$$

$$\ln f(x) = -bx^{-4} \quad (5)$$

$$f(x) = e^{-bx^{-4}}$$

$$f(t) = e^{-bt^{-4}} \quad (2)$$

$$1 + \int_a^x \ln e^{-bt^{-4}} dt = \frac{2}{x^3}$$

$$1 + \int_a^x -bt^{-4} dt = \frac{2}{x^3}$$

$$1 + 2t^{-3} \Big|_a^x = \frac{2}{x^3}$$

$$1 + 2x^{-3} - 2a^{-3} = \frac{2}{x^3} \quad (5)$$

$$\frac{1}{2} = a^{-3}$$

$$a = \sqrt[3]{2} \quad (3)$$

Let $g(x) = \int_3^x f(t) dt$, where f is the function whose graph is shown on the right.

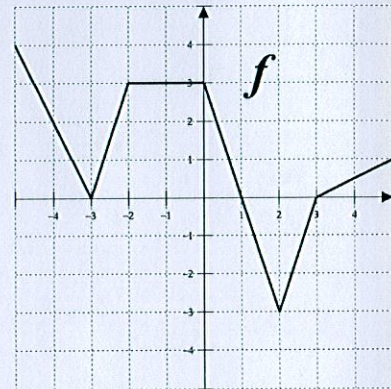
SCORE: ____ / 30 PTS

[a] Find $g(-4)$.

$$\int_3^{-4} f(t) dt = -\int_{-4}^3 f(t) dt$$

$$= - \left[\frac{1}{2}(2)(1) + \frac{1}{2}(2+4)(3) - \frac{1}{2}(3)(2) \right]$$

$$= -7 \quad \textcircled{2} \text{ EACH}$$



[b] Find $g'(-4)$. Explain your answer very briefly.

$$g'(-4) = f(-4) = 2 \quad \textcircled{2\frac{1}{2}}$$

[c] Find all critical numbers of g that do **NOT** correspond to local maxima nor local minima. Explain your answer very briefly.

$$g'(x) = f(x) = 0 \quad \text{BUT DOESN'T CHANGE SIGNS @ } x = -3$$

$$\textcircled{2\frac{1}{2}} \text{ EACH}$$

[d] Find all intervals over which g is increasing and concave down. Explain your answer very briefly.

$$g'(x) = f(x) > 0 \quad \text{AND DECREASING ON } (-5, -3) \quad \text{AND } (0, 1)$$

$$\textcircled{2\frac{1}{2}} \text{ EACH}$$